Electrostatic Field Equations in Dielectrics

The electrostatic equations for fields in **dielectric materials** are:

$$\nabla x \mathbf{E}(\bar{r}) = 0$$

$$\nabla \cdot \mathbf{D}(\bar{r}) = \rho_v(\bar{r})$$

$$\mathbf{D}(\bar{r}) = \varepsilon(\bar{r})\mathbf{E}(\bar{r})$$

In **integral** form, these equations are:

$$\oint_C \mathbf{E}(\bar{r}) \cdot \overline{d\ell} = 0$$

$$\oiint_S \mathbf{D}(\bar{r}) \cdot \overline{ds} = Q_{enc}$$

$$\mathbf{D}(\bar{r}) = \varepsilon(\bar{r})\mathbf{E}(\bar{r})$$

Likewise, for free charge located in some **homogeneous** (i.e., constant) material with permittivity ε, we get the following relations:

$$\mathbf{E}(\bar{\mathbf{r}}) = \frac{Q}{4\pi\varepsilon} \frac{\bar{\mathbf{r}}-\bar{\mathbf{r}}'}{|\bar{\mathbf{r}}-\bar{\mathbf{r}}'|} \quad \text{(for point charge Q)}$$

$$V(\bar{\mathbf{r}}) = \frac{1}{4\pi\varepsilon} \iiint_V \frac{\rho_v(\bar{\mathbf{r}}')}{|\bar{\mathbf{r}}-\bar{\mathbf{r}}'|} \, dv'$$

$$\nabla^2 V(\bar{\mathbf{r}}) = \frac{-\rho_v(\bar{\mathbf{r}})}{\varepsilon}$$

In other words, for homogenous materials, **replace** ε_0 (the permittivity of free-space) with the more general permittivity value ε.

Pretty **simple** !

For **example**:

If the media is **free-space**, use the permittivity of **free-space**.

If the media is, for example, **plastic**, then use the permittivity of **plastic**.